Reg. No.

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.

(For those admitted in June 2024 and later)

PROGRAMME AND BRANCH: B.Sc., STATISTICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART – III	CORE-2	U24ST102	PROBABILITY THEORY

Date & Session: 12.11.2024 / FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** **Answer ALL Questions.**
CO1	K1	1.	How the result of a random experiment will be called? a) Prior b) Trial c) Outcome d) Event
CO1	K2	2.	The set of all possible outcome of a given random experiment is called the_________ associated with that experiment. a) Trial b) Sample space c) Sample Points d) Elementary Event
CO2	K1	3.	A random variable X is said to be ________________if it can take all possible values (integral as well as well a fractional) between certain limits. a) Distinct b) Diverse c) Continuous d) Discrete
CO2	K2	4.	Let X be a random variable. The function F defined for all real x by F(x)=_____ is called the distribution function of the random variable (X). a) $P(X \leq x)$ b) $P(X \neq x)$ c) $P(X > x)$ d) $P(X = 0)$
CO3	K1	5.	Two random variables X and Y with joint probability density function (probability mass function) $f_{XY}(x,y)$ and marginal probability density functions (probability mass functions) $f_X(x)$ and $g_Y(y)$ respectively are said to be stochastically independent if and only if________. a) $f_{XY}(x,y) = f_X(x)/g_Y(y)$ b) $f_{XY}(x,y) = f_X(x) - g_Y(y)$ c) $f_{XY}(x,y) = f_X(x) + g_Y(y)$ d) $f_{XY}(x,y) = f_X(x).g_Y(y)$
CO3	K2	6.	For two dimensional random variable (X,Y). the joint distribution function $F_{XY}(x,y)$ for any real numbers x and y is given by: a) $F_{XY}(x,y) = P(X \leq x, Y \geq y)$ b) $F_{XY}(x,y) = P(X \geq x, Y \geq y)$ c) $F_{XY}(x,y) = P(X \geq x, Y \leq y)$ d) $F_{XY}(x,y) = P(X \leq x, Y \leq y)$
CO4	K1	7.	If X is a random variable. then $V(aX + b) =$ ____ where a and b are constants. a) $a^2V(X)$ b) $aV(X)$ c) $V(a^2X)$ d)$V(aX)$
CO4	K2	8.	If X and Y are independent random variables. then ______________. a) $E(XY) < E(X).E(Y)$ b) $E(XY) = E(X).E(Y)$ c) $E(XY) > E(X).E(Y)$ d) $E(XY) \neq E(X).E(Y)$
CO5	K1	9.	$M_{cX}(t) =$ ___ ,c being a constant. a) $M_X(t)$ b)$M_{-cX}(t)$ c) $M_{cX}(-t)$ d) $M_X(ct)$
CO5	K2	10.	The moment generating function of the sum of a number of independent random variables is equal to the_________ of their respective moment generating functions. a) difference b) divisors c) product d) modulus
Course Outcome	**Bloom's K-level**	**Q. No.**	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K3	11a.	If A and B are any two events (subsets of sample space S) and are not disjoint then construct $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(OR)**
CO1	K3	11b.	Examine the Statement and proof of the Multiplication theorem of probability for Independent events (Two event case).
CO2	K3	12a.	Let X be a continuous random variable with probability density function given by $f(x) = \begin{cases} kx & 0 \leq x < 1 \\ k & 1 \leq x < 2 \\ -kx + 3k & 2 \leq x < 3 \\ 0 & otherwise \end{cases}$ Determine the constant k **(OR)**
CO2	K3	12b.	The diameter of an electric cable, say X, is assumed to be a continuous random variable with probability function:

			$f(x) = 6x(1-x), 0 \le x \le 1$. Show that f(x) is probability density function.
CO3	K4	13a.	Suppose that two dimensional continuous random variable (X,Y) has joint probability density function given by: $f(x,y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$ (i) Examine that $\int_0^1 \int_0^1 f(x,y)\, ds\, dy = 1$ (ii) Identify $P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right)$ **(OR)**
CO3	K4	13b.	A two dimensional random variable (X,Y) have bivariate distribution given by $P(X = x, Y = y) = \frac{x^2+y}{32}, for\ x = 0,1,2,3\ and\ y = 0,1$ Calculate the marginal distribution of X and Y
CO4	K4	14a.	If X and Y are random variable. then examine and prove that $E(X+Y) = E(X) + E(Y)$ provided all the expectations exist **(OR)**
CO4	K4	14b.	If X is a random variable and 'a' is constant, then examine (i) $E[a\varphi(X)] = aE[\varphi(X)]$ (ii) $E[\varphi(X) + a] = E[\varphi(X)] + a$ where $\varphi(X)$, a function of X, is a random variable and all the expectations exist.
CO5	K5	15a.	(i) Show the statement of Bernoulli's law of large numbers (ii) Show the statement of Weak law of large numbers **(OR)**
CO5	K5	15b.	Define the Statement and proof of Additive property of Cumulants.

<table>
<tr><th>Course Outcome</th><th>Bloom's K-level</th><th>Q. No.</th><th>SECTION – C (5 X 8 = 40 Marks)
Answer ALL Questions choosing either (a) or (b)</th></tr>
<tr><td>CO1</td><td>K3</td><td>16a.</td><td>Sketch the Statement and proof of Bayes' Theorem
(OR)</td></tr>
<tr><td>CO1</td><td>K3</td><td>16b.</td><td>Examine the Statement and proof of Boole's Inequality</td></tr>
<tr><td>CO2</td><td>K4</td><td>17a.</td><td>Identify the value of k and then calculate mean, variance, and the coefficients of β_1 of the distribution:
$dF = kx^2e^{-x}\, dx = 1, 0 < x < \infty$
(OR)</td></tr>
<tr><td>CO2</td><td>K4</td><td>17b.</td><td>A random variable X has the following probability function:
<table>
<tr><td>Value of X, x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>p(x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr>
</table>
(i) Identify k
(ii) Calculate $P(X < 6), P(X \ge 6)\ and\ P(0 < X < 5)$</td></tr>
<tr><td>CO3</td><td>K4</td><td>18a.</td><td>Joint distribution of X and Y is given by $f(x,y) = 4xye^{-(x^2+y^2)}; x \ge 0, y \ge 0$.
Test whether X and Y are independent. For the above joint distribution, calculate the conditional density of X given Y=y.
(OR)</td></tr>
<tr><td>CO3</td><td>K4</td><td>18b.</td><td>The joint probability distribution of two random variables X and Y is given by:
$P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$ and $P(X = 1, Y = 1) = \frac{1}{3}$.
Identify (i) Marginal distribution of X and Y
(ii) The conditional probability distribution of X given Y=1</td></tr>
<tr><td>CO4</td><td>K5</td><td>19a.</td><td>Let $X_1, X_2, \ldots . X_n$ be a random variable then prove that
$V\left(\sum_{i=1}^{n} a_iX_i\right) = \sum_{i=1}^{n} a_i^2V(X_i) + 2\sum_{i=1}^{n}\sum_{i=1}^{n} a_ia_jCov\,(X_i, X_j)$
(OR)</td></tr>
<tr><td>CO4</td><td>K5</td><td>19b.</td><td>State and prove Cauchy-Schwartz Inequality</td></tr>
<tr><td>CO5</td><td>K5</td><td>20a.</td><td>Defend Chebychev's Inequality with statement and proof
(OR)</td></tr>
<tr><td>CO5</td><td>K5</td><td>20b.</td><td>(i) Discuss Characteristics function of a random variable and State any three properties of Characteristic function (6 marks)
(ii) Evaluate convergence in Probability (2 marks)</td></tr>
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